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Two-Dimensional Stability of Laminar Flames

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Symbols

A_b	frequency factor for backward reaction
A_f	frequency factor for forward reaction
c_p	specific heat at constant pressure
D	diffusion coefficient
E	activation energy
h_{ct}	heat of combustion of reaction
h_i	enthalpy of species i
h_i^o	heat of formation of species i
$J_{i,j}$	Jacobian of reaction i with respect to species j
k	wave number
Le	Lewis number
M	molecular weight
m, θ	activation parameters
Pr	Prandtl number
p	pressure
R	universal gas constant
Re	Reynolds number
s	stoichiometric ratio
T	temperature
T_r	reference temperature, which is the same as initial temperature
t	time
u	streamwise velocity
v	transverse velocity
w	wavelength
x	streamwise coordinate
Y_i	mass fraction of species i
y	transverse coordinate
z	product of density and velocity u
δ_f	flame thickness
κ	conductivity
μ	laminar viscosity
ρ	density
τ	nondimensional temperature
ϕ	disturbance function
$\dot{\omega}'''$	volumetric reaction rate

Subscripts:

<i>ad</i>	adiabatic
<i>f</i>	flame
<i>i</i>	species: 1, fuel; 2, oxidizer; 3, product; and 4, inert
<i>o</i>	total
<i>r</i>	reference condition
<i>s</i>	steady state

Abstract

This numerical study of the stability of laminar flames specifically addresses the dependence of stability on finite-rate chemistry with low activation energy and variable thermodynamic and transport properties. The calculations show that activation energy and details of chemistry play a minor role in altering the linear neutral stability results from asymptotic analysis. Variable specific heat makes a marginal change to the stability; variable transport properties, on the other hand, tend to substantially enhance the stability from a critical wave number of about 0.50 to 0.20. Also, the effects of variable properties tend to nullify the effects of nonunity Lewis number. When the Lewis number of a single species is different from unity, as is true in a hydrogen-air premixed flame, the stability results remain close to that of unity Lewis number.

Introduction

The linear stability of premixed flames has been studied by a large number of workers. Starting with Landau (1944) and Darrieus (1945), a number of workers, including Margolis, Matkowsky, Sivashinsky, and Clavin (see Clavin 1985 for a list of references), have contributed to this research. A scholarly review appears in Clavin (1985). Landau and Darrieus treated the one-dimensional flame as a thin discontinuity across which a density jump occurs. The dominant parameter that affects the stability is the disturbance wavelength in a direction normal to the direction of propagation. The investigations were conducted to find if such a lateral disturbance of wavelength w (or wave number k) was amplified by the flow field, and the result was that the flame is unstable to all wave numbers. This result was puzzling for a long time because stable laminar flames could be set up in laboratories. Only recently have effects of nonnormal diffusion, curvature, and buoyancy on the flame stability been shown to reflect realistic features of stability. Studies by Sivashinsky, Matkowsky, and others included the effects of nonnormal diffusion (thermal and mass diffusion being unequal), first with low heat release and then with significant heat release. The analyses of Pelce and Clavin (1982), Matalon and Matkowsky (1982), and Frankel and Sivashinsky (1982) consider the limit of wave number tending to zero. The nondimensionalizations used are different in these analyses, and the analysis of Pelce and Clavin is more general. One of the objectives of the three analyses conducted independently was to demonstrate the weak influence of viscosity in relation to conductivity and diffusivity. These studies obtain the dispersion relation (a result of the stability analysis giving the relationship between the growth rate and wave number) as $\omega = a(q)k - b(q, Le)k^2$, where ω is the growth factor, k the wave number, and a and b are coefficients depending on the temperature ratio T_{ad}/T_o and $Le = D\rho c_p/\kappa$, the ratio of mass to thermal diffusivities. The expressions for a and b obtained by Matalon and Matkowsky (1982) as well as Frankel and Sivashinsky (1982) do not match (after appropriate transformations) even for a critical wave number. The numerical differences are not large in the range of parameters of interest, and the values predicted for a critical wave number are the same for $Le = 1$. Results based on systematic analysis and numerical integration of disturbance equations have been made by Jackson and Kapila (1984). Their numerical calculations have spanned the complete range of wave numbers, and they confirm the earlier results in the appropriate limits. They deduce from such analysis the influence of exothermicity and buoyancy on flame stability (Jackson and Kapila 1986). Increase in exothermicity is shown to destabilize the flame, and buoyancy is shown to stabilize the flame.

All these studies are analytical in nature and have treated the high activation energy limit. In these studies the steady-state profiles for temperature, velocity, and mass fractions are

exponential in character and the reaction zone asymptotes to a plane. A typical plot from the calculations of Jackson and Kapila (1984) is shown in figure 1 for the largest parameter of exothermicity of Jackson and Kapila and without buoyancy. The parameter of exothermicity corresponds to a flame temperature six times the cold reactant temperature. The abscissa in figure 1 is $l = \left(\frac{1}{Le} - 1\right)\theta$. In this figure, m and θ are proportional to the activation energy, and E for the finite-rate chemistry models is discussed later. The results obtained are for $E/RT_{ad} \rightarrow \infty$ and for Lewis number not far from unity. It can be seen that at $l = 0$, corresponding to $Le = 1$, the unstable wave numbers are restricted to less than 0.36. And as Le increases, the range of unstable wave numbers increases. Values of Lewis numbers of some species go up to 2 in the case of H_2 -air systems. For $Le = 2.0$ and $E/RT_{ad} = 4$ for the H_2 -air system, one obtains $l = -2$ and $k_{crit} = 0.6$. Thus the unstable wave number is increased substantially. At $Le = 1$, the unstable wavelengths are larger than 18 ($= 2\pi/k_{crit}$) times the flame thickness. The flame thickness is defined in these studies by $\delta_f = \kappa_r/c_{p,r}\rho_r u_r$, where κ_r is the reference conductivity, $c_{p,r}$ is the specific heat, ρ_r is the reference density, and u_r is the reference flame speed. For cold reference conditions, this gives

$$\begin{aligned}\delta_f &\approx \frac{0.052 \text{ W/m-K}}{(1390 \text{ J/kg-K} \times 0.85 \text{ kg/m}^3 \times 1.8 \text{ m/s})} \\ &\approx 0.0000244 \text{ m (0.0244 mm)}\end{aligned}$$

For hot reference conditions, conductivity is almost 3.7 times the cold value and specific heat is 25 percent higher. The product of density and flame speed is constant for the respective cold and hot values, and $\delta_f \approx 0.072 \text{ mm}$. Therefore, the lateral wavelength causing instability is $18\delta_f$, about 1.3 mm. In the above results, the principal controlling factor is hydrodynamics. The role of diffusion seems significant only for Lewis number significantly different from unity.

As stated earlier, in all the above analyses, the activation energy is treated as large. The overall activation energy has been estimated by Fenn and Calcote (1957) to be 16 kcal/g-mole for the H_2 -air system and 28 to 30 kcal/g-mole for many stoichiometric hydrocarbon air systems. At typical flame temperatures of 2300 K, the activation parameter $\theta = E/RT_{ad} \approx 3.50$ for H_2 -air and 6.1 to 6.5 for hydrocarbon-air mixtures. Arguments concerning the validity of asymptotic analysis are made after estimating that θ is between 10 to 20 (Clavin 1985). As noted previously, the perceived values of overall activation energy for equivalent single-step reaction are much smaller. Although this departure may still permit the validity of asymptotic analysis, there needs to be a demonstration of these aspects. Also, one may find that departures are small with regard to a few aspects, whereas for others, depending on the controlling phenomena, they are not.

Present Work

In the present work, the linear stability of flames is investigated numerically with particular reference to a stoichiometric H_2 -air system by using a single-step finite reaction model. Two classes of reaction models, called “model A” and “model B,” are treated. Both are finite distributed reaction models. Model A is chosen because exact analytical solutions may be obtained of the steady state (or mean flow as it is called in stability analyses) and is primarily used to evaluate the effects of activation energy. Model B is in the line of classical single-step reaction models where numerical solutions are needed even for steady state. In this model the effects of variable thermodynamic and transport properties as well as the effects of diffusion are explored in detail. In the single-step reaction, there are four species, namely fuel ($i = 1$), oxidizer ($i = 2$), product ($i = 3$), and inert ($i = 4$). If the diffusion is modeled after the trace diffusion approximation (see Spalding 1957a and 1957b for details), one has four Lewis numbers, Le_i , $i = 1$ to 4, defined by $Le_i = \rho D_i c_p / \kappa$, where D_i is the trace diffusion coefficient. Instead of letting Le_i

vary through the field (something which is easily possible), different diffusion models are chosen to bring out the effects of diffusion explicitly. The first of these takes all the Lewis numbers to be equal to unity. This forms the reference case explored by the other investigators in asymptotic analysis. The second model takes all the Lewis numbers to be equal to 2.0. This value is chosen to represent the Lewis number of the hydrogen fuel in the environment of the other species. The third model takes $Le_1 = 2$ and $Le_i = 1$, $i = 2, 3, 4$. It is denoted by $Le_i = 2111$. This is chosen because in a typical hydrogen-air flame, the Lewis numbers for species other than hydrogen are near unity. In addition, the sensitivity of the results to the accuracy of the steady-state profiles is explored.

Basic Equations

The two-dimensional problem is set into an x - y Cartesian coordinate system, with the steady flame uniform in y and varying along x . A simple step reaction



is assumed. The conservation equations are as follows:

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \quad (1)$$

x -momentum:

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \mu \left(\frac{4}{3} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{3} \frac{\partial^2 v}{\partial x \partial y} \right) \quad (2)$$

y -momentum:

$$\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \mu \left(\frac{4}{3} \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1}{3} \frac{\partial^2 u}{\partial x \partial y} \right) \quad (3)$$

Energy:

$$\rho \frac{\partial h_s}{\partial t} + \rho u \frac{\partial h_s}{\partial x} + \rho v \frac{\partial h_s}{\partial y} = \frac{\partial}{\partial x} \left(\kappa \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\kappa \frac{\partial T}{\partial y} \right) - \sum_{i=1}^{ns} (h_i + h_i^o) \dot{\omega}_i''' \quad (4)$$

Species conservation:

$$\rho \frac{\partial Y_i}{\partial t} + \rho u \frac{\partial Y_i}{\partial x} + \rho v \frac{\partial Y_i}{\partial y} = \frac{\partial}{\partial x} \left(D_{i\rho} \frac{\partial Y_i}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_{i\rho} \frac{\partial Y_i}{\partial y} \right) + \dot{\omega}_i''' \quad (i = 1, 2, 3) \quad (5)$$

$$\frac{\rho T}{M} = \frac{p}{R} \quad (6)$$

where $\dot{\omega}_i'''$ is the reaction rate of i th species. The primes denote the character of volumetric reaction rate. It is to be noted that for the single-step reaction,

$$\dot{\omega}_1''' = \frac{\dot{\omega}_2'''}{s} = -\frac{\dot{\omega}_3'''}{s+1} \quad (7)$$

where s is the stoichiometric ratio and D_i represents the diffusion coefficients chosen in the present study to give the Lewis number of the i th species a desired value. The mass fraction of

the inert species ($i = 4$) is obtained from the identity that the four mass fractions must sum to unity. The equation of state assumes that pressure p is constant and is a good approximation for the stability study as well (Matalon and Matkowsky 1982). The momentum equations ignore the variation of viscosity in the field. This is assumed by noting the already demonstrated weak effects. (See Clavin 1985.) On the other hand, the variation of conductivity is accounted for because it is known to influence the stability characteristics significantly. The equations are nondimensionalized as follows:

$$\left. \begin{aligned} \frac{x}{\delta_f} &= \bar{x} & \frac{y}{\delta_f} &= \bar{y} & \frac{t u_r}{\delta_f} &= \bar{t} \\ \frac{\rho}{\rho_r} &= \bar{\rho} & \frac{T}{T_r} &= \bar{T} & \frac{u}{u_r} &= \bar{u} \\ \frac{v}{u_r} &= \bar{v} & \frac{\kappa}{\kappa_r} &= \bar{\kappa} & \frac{\mu}{\mu_r} &= \bar{\mu} \\ \frac{D_i \rho}{(D\rho)_r} &= \overline{D_i \rho} \end{aligned} \right\} \quad (8)$$

The subscript r refers generally to cold upstream conditions, and δ_f is the flame thickness chosen as $\delta_f = \kappa_r / (\rho_r u_r c_{p,r})$. This implies that the Reynolds number based on the flame thickness is

$$\text{Re} = \frac{\rho_r u_r \delta_f}{\mu_r} = \frac{\kappa_r}{\mu_r c_{p,r}} = \frac{1}{\text{Pr}} \quad (9)$$

Thus this choice of δ_f implies that the product of Reynolds number and Prandtl number of the flame is unity. Another nondimensional number which appears in the equations is Schmidt number

$$\text{Sc} = \frac{\bar{\mu}_r}{(\overline{D\rho})_r}$$

which is set to unity to obtain $(\overline{D\rho})_r$.

Dropping the bars over the symbols gives the following nondimensionalized equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0 \quad (10)$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \left(\frac{4}{3} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{3} \frac{\partial^2 v}{\partial x \partial y} \right) \text{Pr} \quad (11)$$

$$\rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \left(\frac{4}{3} \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1}{3} \frac{\partial^2 u}{\partial x \partial y} \right) \text{Pr} \quad (12)$$

$$\rho \frac{\partial h_s}{\partial t} + \rho u \frac{\partial h_s}{\partial x} + \rho v \frac{\partial h_s}{\partial y} = \frac{\partial}{\partial x} \left(\kappa \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\kappa \frac{\partial T}{\partial y} \right) - \sum_{i=1}^{ns} \frac{h_i + h_i^o}{c_{p,r} T_r} \frac{\delta_f}{\rho_r u_r} \dot{\omega}_i''' \quad (13)$$

$$\rho \frac{\partial Y_i}{\partial t} + \rho u \frac{\partial Y_i}{\partial x} + \rho v \frac{\partial Y_i}{\partial y} = \frac{\partial}{\partial x} \left(D_i \rho \frac{\partial Y_i}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_i \rho \frac{\partial Y_i}{\partial y} \right) + \frac{\delta_f}{\rho_r u_r} \dot{\omega}_i''' \quad (i = 1, 2, 3) \quad (14)$$

$$\rho T = M_s \quad (15)$$

These are the basic equations we need to solve.

Steady-State Equations

Model A

As discussed earlier, the one-dimensional model utilizes a formalism for which exact analytical solutions are available. The steady-state energy equation for model A is written as follows:

$$\frac{d^2\tau}{dx^2} - \frac{d\tau}{dx} = -\frac{h_{ct}\delta_f\dot{\omega}'''}{\rho_r u_r (T_{ad} - 1)} \quad (16)$$

where $\tau = (T - 1)/(T_{ad} - 1)$ and h_{ct} is the nondimensional heat of combustion.

The thermodynamic and transport properties are taken as constant and the Lewis number is taken as unity. The species conservation equation is eliminated by adding to it the energy equation and obtaining the result of constant enthalpy through the flame. The continuity equation reduces to $(\rho u)_s = 1$, and the momentum equation reduces to $p = \text{Constant}$ after ignoring viscous and inertial terms. The choice of the reaction term is discussed in the section on solutions.

Model B

For model B the same approximations as for model A are utilized, and the energy equation is written similarly. The reaction equation and the rate expressions are



$$\dot{\omega}''' = A_f p^3 Y_{\text{H}_2}^2 Y_{\text{O}_2} \exp\left(\frac{-E_f}{RT}\right) - A_b p Y_{\text{H}_2\text{O}}^2 \exp\left(\frac{-E_b}{RT}\right) \quad (18)$$

The choice of the forward rate constants is discussed later. The backward rate constants are chosen to be consistent with the equilibrium constant for the reaction of equation (17). The resulting one-dimensional equations are solved by a code specifically developed for the purpose (Goyal et al. 1988).

Stability Equations

For stability analysis, the independent variables chosen are $z = \rho u$, v , p , T , and Y_i . The various quantities are expanded around steady state, denoted by the subscript s , as

$$\left. \begin{aligned} \rho(x, y, t) &= \rho_s(x) + \rho_f(x)\phi(y, t) \\ z &= z_s(x) + z_f(x)\phi(y, t) \\ u &= u_s(x) + u_f(x)\phi(y, t) \\ v &= 0 + v_f(x)\phi(y, t) \\ p &= p_s(x) + p_f(x)\phi(y, t) \\ T &= T_s(x) + T_f(x)\phi(y, t) \\ Y_i &= Y_{s,i}(x) + Y_{f,i}(x)\phi(y, t) \end{aligned} \right\} \quad (19)$$

In equation (19), $z_s(x) = \rho u_s(x) = 1$ because at steady state the equation of state gives constant mass flow through the flame. The disturbance function ϕ is chosen as

$$\phi = \exp(-i\omega t + ik y) \quad (i = \sqrt{-1}) \quad (20)$$

Note that $i\omega = \omega_r + i\omega_i$ and the sign of ω_r determines the stability of the flame.

The stability equations for x -momentum containing u_f and u_s must be expressed in terms of other quantities. Manipulations of the equation of state and the expression for z give

$$\left. \begin{aligned} u_f &= \frac{T_s}{M_s} z_f + \frac{T_f}{M_s} \\ u_s &= \frac{T_s}{M_s} \end{aligned} \right\} \quad (21)$$

and

$$\rho_f = -T_f \frac{M_s}{T_s^2} \quad (22)$$

where M_s is the molecular weight variation at steady state. Perturbations of molecular weight are ignored in the analysis.

Model A

For model A, the following equations for z_f , p_f , v_f , and T_f are obtained after the substitution of the expansions into equations (10) through (14) and the subsidiary relationships in equations (21) and (22) are used:

$$z'_f - ik \frac{v_f}{T_s} = -i\omega \frac{T_f}{T_s^2} \quad (23)$$

$$p'_f - T_s z''_f + (T_s - 2T'_s) z'_f + (k^2 T_s + T'_s - T''_s) z_f + J(T_s) T'_f = i\omega z_f \quad (24)$$

$$v''_f - v'_f - k^2 v_f - ik p_f = i \frac{\omega v_f}{u_s} \quad (25)$$

$$T''_f - T'_f + [J(T_s) - k^2] T_f - \frac{T'_s}{T_s} u_f = i \frac{\omega T_f}{T_s} \quad (26)$$

The primes denote derivatives with respect to x . This set of equations constitutes a seventh-order system. These are solved under conditions of zero values as $x \rightarrow \pm\infty$ for all the variables z_f , p_f , v_f , and T_f . Since the number of boundary conditions is eight, the problem is overdetermined and has nontrivial solutions for specific values of wave number k . Thus the problem becomes one of eigenvalue.

Model B

For model B, the following perturbation equations for z , v , p , and T are obtained when equations (19) and (20) are used in equations (10) through (14) and the subsidiary relationships in equations (21) are also used:

$$z'_f - ik \frac{v_f}{T_s} = -i\omega \frac{T_f M_s}{T_s^2} \quad (27)$$

$$p'_f - \text{Pr} \left(\frac{u''_f}{3} - k^2 u_f + i \frac{k}{3} v'_f \right) + u'_s z_f + u'_f = i\omega \frac{M_s}{T_s} u_f \quad (28)$$

$$v''_f - \frac{3}{4\text{Pr}} v'_f - \frac{3}{4} k^2 v_f + ik \frac{u_s}{4} z'_f + ik \frac{u'_s}{4} z_f + i \frac{k}{4M_s} T'_f - i \frac{k M'_s}{4M_s^2} T_f - i \frac{3}{4\text{Pr}} k p_f = i\omega v_f \frac{3}{4\text{Pr}} \frac{M_s}{T_s} \quad (29)$$

$$T_f'' + \left(\frac{-\bar{c}_p}{\kappa_s} + \frac{2}{\kappa_s} \frac{d\kappa_s}{dT} T_s' \right) T_f' - \left(k^2 + \frac{1}{\kappa_s} \sum_{i=1}^{ns} \frac{h_i + h_i^o}{c_{p,r} T_r} \frac{\delta_f}{\rho_r u_r} J_{i,T} \right) T_f$$

$$- \frac{1}{k_s} \sum_{i=1}^{ns} \frac{h_i + h_i^o}{c_{p,r} T_r} \sum_{j=1}^{ns} \frac{\delta_f}{\rho_r u_r} J_{i,j} Y_{j,f} - \frac{\bar{c}_p}{\kappa_s} T_s' z_f = -i \frac{M_s}{T_s} \frac{\bar{c}_p}{\kappa_s} \omega T_f \quad (30)$$

$$Y_{i,f}'' - k^2 Y_{i,f} + \left[-\frac{1}{(D_i \rho)_s} + \frac{1}{(D_i \rho)_s} \frac{d(D_i \rho)}{dT_s} T_s' \right] Y_{i,f}' + \frac{1}{(D_i \rho)_s} \frac{\delta_f}{\rho_r u_r} \left(J_{i,T} T_f + \sum_{j=1}^{ns} J_{i,j} Y_{j,f} \right)$$

$$- \frac{1}{D_i \rho_s} Y_{i,s}' z_f \frac{1}{(D_i \rho)_s} \frac{d(D_i \rho)}{dT_s} Y_{1,s}' T_f' = -i \frac{M_s}{T_s} \frac{1}{(D_i \rho)_s} \omega Y_{i,f} \quad (31)$$

where

$$\bar{c}_p = c_{p,s} + T_s \frac{dc_p}{dT_s} \quad (32)$$

$$J_{i,j} = \frac{\partial \dot{\omega}_i'''}{\partial Y_j} \quad J_{i,T} = \frac{\partial \dot{\omega}_i'''}{\partial T} \quad (33)$$

It is possible to relate the Jacobians $J_{i,j}$ and $J_{i,T}$ by invoking the stoichiometric relations between the reaction rates (eq. (7)) to obtain

$$sJ_{1,j} = J_{2,j} \quad - (s+1)J_{1,j} = J_{3,j} \quad sJ_{1,T} = J_{2,T} \quad - (s+1)J_{1,T} = J_{3,T} \quad (34)$$

In treating the variation of properties, it is assumed that all the dependence of the thermodynamic and transport properties in the flow field is described in terms of temperature alone. This is not entirely correct since there is some dependence of mass fractions of various species as well. But for premixed mixtures, it is reasonably accurate, certainly at a Lewis number of unity, where all the properties are described in terms of one progress variable, namely temperature. For nonunity Lewis numbers, the approximation implies that the extra dependence on mass fractions is ignored. The equation for pressure is in terms of u_f and its derivatives. They can be expressed in terms of z_f and T_f by using relationships in equations (21) to obtain an equation for p_f given by

$$p_f' - \frac{\text{Pr}}{3} \left[4u_s'' z_f + 8u_s' z_f' + 4u_s z_f'' + \frac{4}{M_s} T_f'' - 8T_f' \frac{(M_s')^2}{M_s} - 4 \frac{M_s''}{M_s^2} T_f + 8 \frac{(M_s')^2}{M_s^3} T_f \right.$$

$$\left. - 3k^2 u_s z_f - 3k^2 \frac{T_f}{M_s} + ikv_f' \right] + 2u_s' z_f + u_s z_f' + \frac{1}{M_s} T_f' - \frac{T_f}{M_s^2} M_s' = i\omega z_f + i\omega \frac{T_f}{T_s} \quad (35)$$

The primes on various quantities represent derivatives with respect to x . The perturbation equations require u_s , derivatives u_s (up to the second order), first derivatives of T_s and Y_i , and Jacobians of reaction rate with respect to T and Y_i .

The order of the equations can be reduced further. One should normally solve three species conservation equations. However, the perturbation on the summation of mass fractions leads to

$$Y_{1,f} + Y_{2,f} + Y_{3,f} = 0 \quad (36)$$

Hence it is sufficient to solve only two of the species conservation equations. The energy and species equations must be recast with the relationships given in equation (7) to get

$$T_f'' + \left(\frac{-\bar{c}_p}{\kappa_s} + \frac{2}{\kappa_s} \frac{d\kappa_s}{dT} T_s' \right) T_f' - \left(k^2 + \frac{1}{\kappa_s} h_{ct} \frac{\delta_f}{\rho_r u_r} J_{1,T} \right) T_f - \frac{1}{\kappa_s} h_{ct} \frac{\delta_f}{\rho_r u_r} \left[(J_{1,1} - J_{1,3}) Y_{1,f} + (J_{1,2} - J_{1,3}) Y_{2,f} \right] - \frac{\bar{c}_p}{\kappa_s} T_s' z_f = -i \frac{M_s}{T_s} \frac{\bar{c}_p}{\kappa_s} \omega T_f \quad (37)$$

$$Y_{1,f}'' + \left[-k^2 + \frac{1}{(D_1\rho)_s} \frac{\delta_f}{\rho_r u_r} (J_{1,1} - J_{1,3}) \right] Y_{1,f} - \frac{1}{(D_1\rho)_s} \frac{\delta_f}{\rho_r u_r} (J_{1,2} - J_{1,3}) Y_{2,f} - \frac{1}{(D_1\rho)_s} Y_{1,f}' - \frac{1}{D_1\rho_s} Y_{1,s}' z_f + \frac{1}{(D_1\rho)_s} \frac{\delta_f}{\rho_r u_r} J_{1,T} T_f' + \frac{1}{(D_1\rho)_s} \frac{d(D_1\rho)}{dT_s} T_s' Y_{1,f}' + \frac{1}{D_1\rho_s} \frac{d(D_1\rho)}{dT_s} Y_{1,s}' T_f' = -i \frac{M_s}{T_s} \frac{1}{(D_1\rho)_s} \omega Y_{1,f} \quad (38)$$

$$Y_{2,f}'' + \left[-k^2 + \frac{1}{(D_1\rho)_s} \frac{\delta_f}{\rho_r u_r} (J_{2,1} - J_{2,3}) \right] Y_{2,f} - \frac{1}{(D_2\rho)_s} \frac{\delta_f}{\rho_r u_r} (J_{2,1} - J_{2,3}) Y_{1,f} - \frac{1}{(D_2\rho)_s} Y_{2,f}' - \frac{1}{(D_2\rho)_s} Y_{2,s}' z_f + \frac{1}{(D_2\rho)_s} \frac{\delta_f}{\rho_r u_r} J_{1,T} T_f' + \frac{1}{(D_2\rho)_s} \frac{d(D_2\rho)}{dT_s} T_s' Y_{1,f}' + \frac{1}{D_2\rho_s} \frac{d(D_2\rho)}{dT_s} Y_{2,s}' T_f' = -i \frac{M_s}{T_s} \frac{1}{(D_2\rho)_s} \omega Y_{2,f} \quad (39)$$

In obtaining these equations, the following equalities obtained from equations (34) have been used:

$$J_{2,2} - J_{2,3} = s(J_{1,2} - J_{1,3}) \quad J_{2,1} - J_{2,3} = s(J_{1,1} - J_{1,3}) \quad (40)$$

$$h_{ct} = \frac{h_1 + h_1^0 + s(h_2 + h_2^0) - (1+s)(h_3 + h_3^0)}{c_{p,r} T_r} \quad (41)$$

If the choice of Lewis numbers is such that $Le_2 = Le_3$, then the two species equations can be related by $sY_{2,f} = -(s+1)Y_{3,f}$. This equation can be used to reduce the number of species equations to one and to modify the energy equation. Such a modification gives

$$T_f'' + \left(\frac{-\bar{c}_p}{\kappa_s} + 2\kappa_s \frac{d\kappa_s}{dT_s} T_s' \right) T_f' + \left(-k^2 + \frac{1}{\kappa_s} h_{ct} \frac{\delta_f}{\rho_r u_r} J_{1,T} \right) T_f - \frac{1}{\kappa_s} h_{ct} \frac{\delta_f}{\rho_r u_r} J_\sigma Y_{1,f} - \frac{\bar{c}_p}{\kappa_s} T_s' z_f = -i \frac{M_s}{T_s} \frac{\bar{c}_p}{\kappa_s} \omega T_f \quad (42)$$

$$Y_{1,f}'' + \left(-k^2 + \frac{1}{D_1\rho_s} \frac{\delta_f}{\rho_r u_r} J_\sigma \right) Y_{1,f} + \left[\frac{-1}{(D_1\rho)_s} + \frac{1}{(D_1\rho)_s} \frac{dD_1\rho}{dT_s} \right] T_s' Y_{1,f}' - \frac{1}{(D_1\rho)_s} Y_{1,s}' z_f + \frac{1}{(D_1\rho)_s} \frac{\delta_f}{\rho_r u_r} J_{1,T} T_f' + \frac{1}{(D_1\rho)_s} \frac{dD_1\rho}{dT_s} Y_{1,s}' T_f' = -i\omega \frac{M_s}{T_s} \frac{1}{(D_1\rho)_s} Y_{1,f} \quad (43)$$

where

$$J_\sigma = J_{1,1} + sJ_{1,2} - (1+s)J_{1,3} \quad (44)$$

In this case the order of the equation to be solved is 9. If $Le_i = 1$, one can eliminate the equation for Y_1 by combining it with the energy equation. The perturbation equation for enthalpy has zero for the solution. For this case, $(D_1\rho)_s = \frac{\kappa_s}{\bar{c}_p}$ and the temperature gradient terms associated with the transport properties are ignored. In this case the energy equation becomes

$$T_f'' - \frac{\bar{c}_p}{\kappa_s} T_f' - T_f \left[k^2 + \frac{1}{\kappa_s} \frac{\delta_f}{\rho r u r} (J_\sigma - h_{ct} J_{1,T}) \right] - \frac{\bar{c}_p}{\kappa_s} \frac{dT_s}{dx} z_f = -i \frac{M_s}{T_s} \frac{\bar{c}_p}{\kappa_s} \omega T_f \quad (45)$$

Steady-State Solutions

Model A

Because equation (16) does not have the space coordinate explicitly, it is possible to reduce the order of the equation by defining $q = \frac{d\tau}{dx}$ (Spalding 1957a and 1957b). One can then recast the equation as

$$q \left(\frac{dq}{d\tau} - 1 \right) = -\Lambda \dot{\omega}''' \quad (46)$$

where Λ represents the constants on the right-hand side of equation (46). Equation (46) has been analyzed in combustion literature. The reaction rate expression starts from exponentially small values near $\tau = 0$, peaks at some value of τ depending on the activation energy, and goes to 0 at $\tau = 1$. Similarly, q is 0 both at $\tau = 0$ and 1 and is positive definite over the range $\tau = 0$ to 1 for the adiabatic case considered here. Based on these observations one can show that, for a class of profiles $q = \tau - \tau^m$, where m is a parameter, one obtains $\Lambda = 1$ and $\dot{\omega}''' = m\tau^m(1 - \tau^{m-1})$. Reversing this argument, one can say that for this reaction rate expression (with m as a parameter), the solution for q is as stated earlier. One can integrate the equation for q and set out the steady-state solution as

$$\tau_s = 1 + \exp[-(m-1)x + c]^{-\frac{1}{m-1}} \quad (47)$$

$$T_s = \tau_s(T_{ad} - 1) + 1 \quad (48)$$

$$u_s = T_s \quad (49)$$

$$\frac{dT_s}{dx} = (T_{ad} - 1) \frac{d\tau_s}{dx} = (T_{ad} - 1) (\tau_s - \tau_s^m) \quad (50)$$

$$\dot{\omega}''' = m\tau_s^m (1 - \tau_s^{m-1}) \quad (51)$$

$$J = \frac{\partial \dot{\omega}'''}{\partial T} = (T_{ad} - 1) m^2 \tau_s^{m-1} \left[1 - \left(2 - \frac{1}{m} \right) \tau_s^{m-1} \right] \quad (52)$$

In these equations, the steady-state result that $(\rho u)_s = 1$ along with the equation of state is used to obtain equation (49). In equation (47), c is chosen so that $\tau = 0.5$ at $x = 0$; this gives

$$c = \log(2^{m-1} - 1) \quad (53)$$

These solutions are coded and used in the solution of the stability equations. The choice of c has no effect except on the resolution of the eigen solutions. With the stability code using a grid distribution which allows a finer resolution at the center ($x = 0$) and increasingly coarse grid at x removed from this point, one expects better resolution by arranging the steady-state solution in this manner. The stability code utilizes its own grid distribution and computes the various quantities with the analytical expressions noted above. The parameter m characterizes

essentially the activation energy; that is, $m = \text{Constant}(E/RT_{\text{ad}})$. Typically values of $m \approx 4$ to 6 imply high activation energy and $m \approx 1.3$ implies activation energies close to 16 kcal/mole of a range expected for a H_2 -air system. This fact is based on the result that the reaction rate distribution (eq. (51)) has a peak at τ_s close to 0.5, a feature seen later even with a reaction rate distribution with τ in the case of full chemistry.

Model B

The numerical solutions for the assumed reaction rate for the H_2 -air system are obtained from an unsteady code developed for the purpose (Goyal et al. 1988). The code also generates the Jacobians of reaction rate with respect to temperature and mass fractions of species to be used in the stability code. These steady-state results are used in the stability calculations for nonunity Lewis number cases. The steady problem uses a uniform grid in $z = \int \rho \, dx$. The grid is then transformed to the coordinate $x = \int dz/\rho$. Then, the results of temperature, mass fractions, and Jacobians are interpolated with a cubic spline interpolation program into the grid required by the stability code. The temperature data then are spectrally differentiated by a Chebyshev polynomial fit to obtain the first and the second derivatives. These profiles were found to be jagged and nonsmooth. Consequently, it was decided to curve fit those data which needed to be differentiated. A Pade polynomial fit was used to describe the temperature distribution with x , and molecular weight, specific heat, conductivities, and diffusivities with temperature. These were then used in the stability code.

Stability Solutions—Numerical Aspects

The stability code used here was originally written for analyzing the stability of high-speed flows (Macaraeg, Streett, and Hussaini 1988). The perturbation equations are discretized by a spectral collocation technique using Chebyshev polynomials as basis functions. The code utilizes a staggered mesh to treat pressure. The resulting discretized equations are written in a generalized matrix eigenvalue problem and are solved with the standard library routine. (See Macaraeg, Streett, and Hussaini 1988.)

Model A

For model A, all the steady-state quantities were known in analytical form, and the calculations of the stability could be performed in a straightforward manner. The code utilized a grid stretching with the finest portion of the grids at $x = 0$. The region covered is from $-\infty$ to ∞ . It was therefore necessary to set a value for infinity. Several initial experiments suggested that infinity could be set at $x = \pm 15$. Sometimes, the eigenfunction could not be resolved accurately, since the decay was slow; for this reason, infinity was set at -20 to $+20$. (It must be remembered that this x is already nondimensionalized by δ_f .) Grid resolution studies were conducted and these showed that the results did not differ by more than 0.1 percent when the number of grid points exceeded 121. Most calculations utilized at least 121 grid points. An interesting aspect of the eigenfunction distribution was that pressure perturbations decayed the slowest toward the boundaries. Initial concerns regarding the effect on accuracy were resolved when it was determined that enhancing the boundaries and increasing the grid resolution did not affect the critical neutral wave number but altered the eigenfunctions marginally.

Model B

For model B, the range of infinity and the grid resolution used for model A were found to be valid. In the numerical results of steady flames, it was necessary to define a value of δ_f . Although it would be possible to estimate the value from $\delta_f = k_r/\rho_r u_r c_{pr}$, it was found convenient to assign a value to δ_f , and with this value, obtain a consistent set of reference values. It should be remembered that the critical wave number, a result from the stability code, is actually a

nondimensional quantity, the nondimensionalizing parameter being δ_f . One would expect the physical results obtained to be independent of the choice of δ_f . This was ensured by varying the value of δ_f and obtaining the critical wavelength for each case.

Results and Discussion

Model A

Figure 2 shows the steady-state profile of temperature, the first gradient, the second gradient, and the Jacobian. Most of the region of large change is restricted to a region $-6 \leq x \leq +6$. The Jacobian varies significantly over the field, and the variation is different for the activation energy parameter $m = 9$ and 2. The variation is larger for $m = 9$ and smaller for $m = 2$. The calculations lead to a set of critical wave numbers (zero growth rate $\omega_i = 0$) for m , the activation energy parameter, as shown in table 1. The peaks of the eigenfunctions are shown in table 1(b) for the unperturbed (U) and the perturbed (P) cases, which are discussed later. It can be seen from the table that the critical wave number varies from 0.36 at high m to about 0.40 at $m \approx 1.3$ corresponding to $E \approx 16$ kcal/mole. This constitutes a 10-percent change which is not considered significant. The eigenfunctions are consistent with results from the asymptotic analysis. The imaginary part of z_f , real part of v_f , and imaginary parts of T_f and p_f are zero. The other nonzero eigenfunctions are normalized by the peak of p_f .

In studies of stability with strong convection such as mixing or boundary layers, it was found that the mean profile exerts a significant influence on the stability characteristics. In order to determine the validity of this statement in the present context and to determine the features which affect stability significantly, subsidiary calculations were performed as follows. The initial profile of $T'_s(x)$ and $T''_s(x)$ was perturbed by a function $4x(x_\infty - x)/x_\infty^2 \sin \frac{3\pi}{2} \frac{x}{x_\infty}$ chosen arbitrarily so that there would be fluctuations in the profile with zero at the boundaries $x = 0$ and $x = x_\infty$. Figure 3 shows the plots of steady profiles. As can be seen, profiles for both dT/dx and d^2T/dx^2 have considerable fluctuations. Table 1 shows k_{crit} and peak amplitudes of eigenfunctions for $m = 9$ and 2. The wave number k_{crit} is altered by no more than 3 percent, and the eigenfunctions are altered somewhat more but less than 10 percent. There are considerable fluctuations in the resulting eigenfunctions, largely those for pressure. These fluctuations do not seem to affect the overall result on stability. Thus the errors in temperature profile gradients seem to make little difference to the results of stability. The reason for this is that the instability is largely driven by hydrodynamics and details of the profile do not matter significantly. Figure 4 shows the eigenfunctions for both low and high activation energies, both perturbed and unperturbed cases. First, consider the unperturbed case. The structure of the eigenfunctions shows that their width is also from -6 to 6 . It is only the eigenfunction for pressure that seems to decay slowly. For the lower activation energy, the temperature eigenfunction peak is larger than the pressure eigenfunction. This feature of the temperature eigenfunction having a peak higher than the pressure eigenfunction is seen in all the later calculations for model B. Between these unperturbed and perturbed cases the effect of disturbance is less severe for $m = 9$ than for $m = 2$. Calculations were made by changing the Jacobian by 5 percent from the nominal value. This results in a substantial change in the critical wave number of 20 percent. The features concerning the eigenfunctions look very similar and seem altered quantitatively to a small extent. Thus the stability is very sensitive to the Jacobians but quite insensitive to the details of temperature profile gradients.

The effects of Prandtl number have been discussed by earlier investigators (Clavin 1985) and were deduced to be insignificant. The results of the dependence of the critical wave number on Prandtl number are presented in table 2. The changes of k_{crit} near $\text{Pr} = 1$ are marginal. Only in the extreme case of $\text{Pr} = 0.05$ does the change of k_{crit} from that of $\text{Pr} = 1$ look substantial. A study that considered $\text{Pr} \rightarrow 0$ was conducted to determine if the viscous terms

could be neglected altogether. Two calculations were made by dropping the viscous terms in the momentum equations for u and v separately. Neglecting the viscous terms in the equation for u for $m = 2$ leads to a 10-percent reduction of k_{crit} from 0.391 to 0.356. Neglecting viscous terms in the equation for v does not lead to an acceptable solution satisfying the boundary conditions. This situation is inferred to be related to the neglect of the highest order derivatives in v (v'' term), which is a typical singular perturbation problem. This is why the approach of obtaining the limiting solution of letting $\text{Pr} \rightarrow 0$ by retaining all the derivatives seems to lead to a physically consistent result.

Model B

Numerical calculations for a steady flame were performed for the stoichiometric H_2 -air system with a single step reaction scheme, $2\text{H}_2 + \text{O}_2 = \text{H}_2\text{O}$, with frequency factor A_f of 1.1×10^{19} , and an activation energy E of 16 kcal/mole. The choice of the parameters was based on the calculations of the stoichiometric flame structure with full chemistry (Bhashym et al. 1986). Figure 5 shows the plot of reaction rate of hydrogen with nondimensional temperature from such a calculation. The peak in the reaction rate occurs at $T \approx 4.2$, whereas the adiabatic temperature corresponds to $T = 8.156$. For $\text{Le} = 1$, the reaction rate expression becomes a function depending on temperature alone. Now, one can estimate E (or θ) from the plot of reaction rate with temperature. Such a calculation yields $E \approx 16$ kcal/mole. Such estimates are also available from earlier work (Fenn and Calcote 1953).

The steady flame speeds obtained from the steady-state calculations are 1.63 ($\text{Le}_i = 1$), 1.83 ($\text{Le}_i = 2$), and 1.70 m/s ($\text{Le}_i = 2111$). The case $\text{Le}_i = 2111$ implies that the Lewis numbers for the four species 1, 2, 3, and 4 are 2, 1, 1, and 1. The results of the steady profiles and the eigenfunctions for the nominal case are shown in figure 6. The critical wave numbers for chosen values of δ_f are shown in table 3. As can be seen, the critical unstable wavelength is about 0.9 mm for the classical constant property case. For large activation energy, the critical wavelength would be about 1.05 mm (not shown in the tables).

The calculations with variable properties show results which are interesting. Variable properties seem to act as a stabilizing influence, raising the unstable wavelength to as large as 1.88 mm. The property variation that has caused the change is deduced from the next two results. Variable specific heat alone seems to slightly destabilize the flame. But conductivity and diffusivity variation coupled through the $\text{Le}_i = 1$ assumption is the most stabilizing feature. It enhances the stability by a factor of 3. Clavin (1985) invoked the work of Clavin and Garcia (1983) and has indicated that the variable property effects can be taken into account by the use of thermal diffusivity at the hot condition rather than the unburnt condition. This effectively amounts to taking δ_f about 2 to 2.5 times higher than that estimated from the use of properties at unburnt condition. This effect then leads to enhanced stability. The results obtained in the current work are in conformity with the results of Clavin. The details can be understood by examining the results set out in figures 7 and 8. As can be seen from these figures, there are only weak differences in the profiles of the eigenfunctions, though the critical value of the wave number is significantly different between the constant and variable property case.

Results of the kind described for model reaction were again established in the present case: (1) an increase of dT/dx by 1.5 changes the predicted critical wavelength by 2 percent, (2) a change in d^2T/dx^2 affects the results even less than a change in dT/dx , and (3) an increase of the Jacobian profile by 10 percent causes an increase in critical wave number of 25 percent (these results are not presented here).

Once the range of infinity -15 to 15 and the scheme for interpolation were established, the approach to curve fit the steady-state quantities was abandoned in favor of numerically differentiating the temperature profile and using other interpolated quantities directly. Calculations

for constant properties turned out to be straightforward and gave the results within 1 percent of those from the curve fit noted above. The results for these calculations are presented in figure 8. The second gradient alone seems to have fluctuations; but this does not seem to affect the eigenfunctions at all. The calculations for variable specific heat turned out to be more difficult to perform and needed better resolution. This was accomplished with 151 grid points. (The CPU times for the calculations of the eigen spectrum and the eigenfunctions on a CRAY-2 super computer were 71 s for 121 grid points and 120 s for 151 grid points for one case).

Calculations have been made for two cases of nonunity Lewis number. In the first case, the Lewis number of all the species was 2. This corresponds to the conventional approach in which all Lewis numbers are equal. In the second case, the Lewis number of fuel alone is taken as 2.0 and the Lewis numbers of the other species were set to unity. This follows from the calculations of H₂-air either with full chemistry or single step chemistry with variable properties (see, for instance, Bhashyam et al. 1986) which show that Le_i for H₂ is about 2 to 2.5, Le_i for others is between 0.8 to 1.0. The steady-state profiles of $Y_{i,s}$ versus T and the Jacobians are shown in figure 9. The profile shapes for $Le_i = 2$ show significant deviations from a linear profile. This is expected from simple analyses of the variation of nondimensional temperature with fuel mass fraction near a cold boundary (Spalding 1957a and 1957b). The profiles for $Le_i = 2111$ (Lewis number of various species in order of H₂, O₂, H₂O, N₂), however, do not differ much from results with $Le = 1$. The differences in the results between $Le_i = 2111$ and $Le = 1$ are caused by the diffusion terms. The stability results are summarized in table 4. The critical wavelength is typically 1.6 to 1.8 mm for the nonunity Lewis number cases. These values are only slightly smaller than for $Le = 1$. These observed features are a consequence of the fact that hydrodynamics controls stability and details of flame structure are less relevant to stability.

Figure 10 shows the plots of the real and imaginary parts of the eigenvalues as a function of wave number k . It may be noted that ω_r is less than 0 for the stable range shown in the figure. In all the cases except $Le_i = 2$ (with variable properties), the imaginary part (ω_i) is zero in the unstable range. The imaginary part being zero implies that the solution gets amplified in a nonoscillatory manner. The growth of the disturbance in time for any given unstable wavelength can be estimated from the results of figure 10. The time for doubling disturbance amplitude can be obtained from the disturbance equation (eq. (16)) as

$$t_{21,s} = \frac{0.693}{\omega_r(k)} \frac{\delta f}{u_r} \quad (54)$$

The time for doubling the amplitude scales like the characteristic time for the flame, with the coefficient typically being about 5 to 20. These values are relevant when making a full nonlinear simulation with a disturbance.

Summary of Results

The problem of the stability of laminar flames, particularly the H₂-air system has been studied. The effects of finite-rate kinetics and variable thermodynamic and transport properties are explored. The perturbation equations are spectrally discretized and numerically solved to obtain the eigenvalues and the corresponding eigenfunctions. These calculations show the following results:

1. The effect of finite activation energy on the critical wavelength is not significant. Reduction of the activation energy to values corresponding to the H₂-air system reduces the critical wavelength by about 10 percent.
2. Variable transport properties enhance the stability and enhance the critical wavelength by a factor of 2 to 2.5.

3. Results for realistic parameters show that the critical unstable wavelength for a stoichiometric H_2 -air mixture is about 1.6–1.8 mm.

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Table 1. Model A

(a) Critical wave number k_{crit}

m	k_{crit} for—	
	Unperturbed case	Perturbed case
9	0.362	0.3704
4	0.375	0.382
2	0.391	0.407
1.3	0.401	0.411

(b) Peak amplitudes of eigenfunctions

m	Case	$\text{Re}(z)$	$\text{Im}(v)$	$\text{Re}(T)$	$\text{Re}(p)$
9	Unperturbed	2.9	17.5	74.0	210.0
9	Perturbed	3.3	19.2	79.1	225.6
2	Unperturbed	8.2	28.1	78.3	58.3
2	Perturbed	9.1	29.3	79.7	62.5

Table 2. Model B

Pr	k_{crit} for m of—	
	9	2
1.0	0.391	0.362
0.7	0.377	0.374
0.1	0.305	0.331
0.05	0.300	0.301

Table 3. Model B for $\text{Le}_i = 1$ and $\theta = 3.5$ (a) Critical wave number k_{crit} and δ_f

Case	k_{crit}	δ_f , mm	Wavelength, $2\pi\delta_f/k_{\text{crit}}$, mm
Constant properties	0.42	0.06	0.9
Variable properties	0.20	0.06	1.88
Variable c_p	0.43	0.06	0.88
Variable κ and $D\rho$	0.18	0.06	2.09

(b) Peak amplitudes of eigenfunctions

Case	$\text{Re}(z)$	$\text{Im}(v)$	$\text{Re}(T)$	$\text{Re}(p)$
Constant properties	10.0	26.0	83.8	64.5
Variable properties	8.0	25.0	74.4	66.5

Table 4. Model B for $\text{Le}_i = 2$ and 2111(a) Critical wave number k_{crit} and δ_f

Case	Le_i	k_{crit}	δ_f , mm	Wavelength, $2\pi\delta_f/k_{\text{crit}}$, mm
Constant properties	2	0.50	0.06	0.75
Variable properties	2	0.24	0.06	1.57
Constant properties	2111	0.48	0.06	0.785
Variable properties	2111	0.21	0.06	1.79

(b) Peak amplitudes of eigenfunctions

Case	Le_i	$\text{Re}(z)$	$\text{Im}(v)$	$\text{Re}(T)$	$\text{Re}(p)$
Constant properties	2	9.5	23.0	86.4	60.4
Variable properties	2	6.2	21.0	84.4	64.5

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